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Optimum shunt capacitor placement in distribution system—A review and comparative study

M.M. Aman [a,*], G.B. Jasmon [a], A.H.A. Bakar [b], H. Mokhlis [a,b], M. Karimi [a]

[a] Department of Electrical Engineering, Faculty of Engineering, University of Malaya, 50603 Kuala Lumpur, Malaysia
[b] UM Power Energy Dedicated Advanced Centre (UMPEDAC) Level 4, Wisma R&D, University of Malaya, Jalan Pantai Baharu, 59990 Kuala Lumpur, Malaysia

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ABSTRACT

Shunt capacitors are commonly used in distribution system for reactive power compensation. Different analytical, numerical programming, heuristic and artificial intelligent based techniques have been proposed in the literature for optimum shunt capacitor bank (SCB) placement. This paper will present a very detailed overview of optimum SCB placement techniques. Six different approaches of optimum SCB placement based on minimization of power losses, weakest voltage bus approach and maximization of system loadability will be applied on four different radial distribution test systems. The results will be compared on the basis of power loss reduction, voltage profile improvement, system loadability maximization and the line limit constraint.

Contents

1. Introduction

Prior to 1950s the shunt capacitor banks (SCB) were placed nearer to the main substation for capacitive reactive power compensation, it helps in improving the power factor, reduces I^2R power losses and improving the voltage profile. SCB changes the power losses up to the point of coupling, however to get the maximum benefit it must be placed as nearer to the load as possible. With the availability of pole mounted equipment including SCB, the trend has changed. The capacitor banks are now placed on primary distribution lines as well [1–5].

The capacitor unit is considered as the basic building block of SCB. Capacitor units are connected in paralleled-series combinations and

* Corresponding author. Tel.: +60 3 79675238; fax: +60 3 79675316.
E-mail address: mohsinaman@gmail.com (M.M. Aman).

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form a single-phase capacitor bank, within a steel enclosure. The series combination reduces the cost of dielectric while parallel combination increase the total capacitance of SCB. As a general rule, the minimum number of units connected in parallel is such that isolation of one capacitor unit in a group should not cause a voltage unbalance more than 110% of rated voltage on the remaining capacitors of the group. Equally, the minimum number of series connected groups is that in which the complete bypass of the group does not subject the others remaining in service to a permanent overvoltage of more than 110% [2]. The amount of reactive power (Q_C) from capacitor depends on applied voltage (V) and capacitive reactance (X_C), given by Eq. (1) [6].

$$Q_C = V^2/X_C \tag{1}$$

The recent power system blackouts [7,8] due to insufficient reactive power have also resulted in focused towards meeting reactive power demand of the system locally using static capacitor banks. The combined US Canada task force on August 2004 blackout also concluded that the reactive power supplies in Northeast Ohio were exhausted which resulted in loss of several critical bulk power supply systems and helped cascaded generator interruptions [9]. During low voltage emergencies e.g. generator rescheduling, line restoration or operated directed load tripping, the author in [10,6] proposed shunt capacitor bank series group shorting (CAPS) method. CAPS shorted section increases the reactive power supplied during periods of low voltages by shorting several series groups of capacitor units ($Q_C\uparrow = V^2/X_C\downarrow$). The shorted section in CAPS comprises of 20% to 33% of total bank. The detailed study and feasibility of CAPS on EHV and HV network are also presented in [11]. In case of highly loaded systems, it is believed that the optimum capacitor placement solve the minimization of losses more adequately and optimum setting of voltage regulators solve the voltage drop problems in a better manner [3].

The need for reactive power support in distribution system may be arises due to the following reasons.

1.1. Maximizing the distribution system efficiency

Distribution system usually suffers from two major problems, high power losses and poor voltage profile. Losses are defined as the difference between the energy into the system and the energy that is utilized by the end users. Generally electric system losses can be categorized as technical or non-technical losses [12,13]. Technical losses in distribution system occurs at different stages from the main substation till the consumer end, including substation transformer, primary lines, line equipment voltage regulators and surge arrestor, distribution transformer, secondary lines and consumer services. The loss calculation methods at different stages are discussed in detail in [14]. Electric Power Research Institute (EPRI) and Energy Information Administration (EIA) of U.S. concluded that [14,15]:

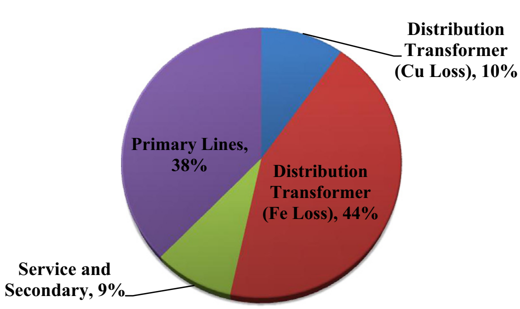

Fig. 1. Breakdown of distribution losses—EPRI study [14].

1. The distribution losses range from 33.7% to 64.9% of the total system losses.
2. About 7% of the total electricity production is transmitted in the United States as transmission and distribution losses [15].
3. EPRI research on 42 distribution circuits estimated that 54% of total losses are occurring in distribution transformer (although the efficiency of distribution transformer lies above 99%) and 38% of total distribution losses are occurring in primary lines, as shown in Fig. 1. One of the major reason for such a high losses in distribution transformer is total number of distribution transformer placed in distribution system [14]. In 2003, it is estimated that there are 50 million distribution transformers in use in United States [16].

With such a higher power losses in distribution system, it is highly necessary to reduce the line losses occurring in primary line as much as possible. The higher losses results in limiting the line capacity (*thermal limits*) as well as higher voltage drop (*voltage limits*) in the power system. In literature it has also been concluded that the maximum loading of the distribution system is limited by the voltage limit rather than the thermal limit [17]. Large power consumers also installed shunt capacitor to improve the overall power factor and thus save the cost of poor power factor penalty. Three different compensation techniques are available in literature including individual compensation; group compensation and centralized compensation to improve the power factor. Any one of the method or all of the method can be utilized to take the maximum advantage of improved power factor [18]. For power factor improvement, an unloaded synchronous motor can also be used instead of shunt capacitor. The amount of reactive power is controlled from its excitation system and thus it behaves like a variable capacitor [19]. Now-a-days, manufacturers are bound to design electrical equipment with higher efficiency and high power factor [20].

1.2. Reactive power management in deregulated power market

With the restructuring of power system, the complexity of power system has been increased. The vertically integrated power system has been separated into GenCOs, TransCOs, and DisCOs. A central regulating company Independent System Operator (ISO) and Regional Transmission Organizations (RTO) has been formed for maintaining the quality, reliability and security of electric service [21,22]. It is also a fact that the existing transmission systems in most of the countries are quite old. For example in United States, the 345 kV bulk transmission system and associated substation, cables and wires are 40 years old and above [23]. Such a system is not able to meet the growing demand and transfer the generated power from the centralized generation to the distribution system. Transmission investment has been falling for a quarter century at an average rate of almost US$50 million a year (in constant 2003 U.S. dollars), however there has been a small upturn in the last few years [23]. Other than constructing new transmission lines there are other options to release the transmission system congestion including distributed generator placement, static capacitor bank, FACTS (Flexible AC Transmission Systems) devices, voltage regulators and energy conservation [24,25]. It is also a fact that the generation the addition of extra kVAR on a generator operating at 0.9 power factor decreases the amount of real power output by about half a kilowatt [26]. Thus in restructured power market, the generator companies prefer to generate maximum active power (kW) and get maximum profit ($).

The reactive power (kVAR) market is not as simple as real power (kW) market. The fundamental difficulty with reactive power markets is that reactive power does not "travel" far, thus it is expected there would be extreme local geographical market

power in the provision of reactive power [27]. This philosophy of reactive power market suggests the following things [26]:

1. The generator should maintain close to unity power factor under normal conditions using capacitor bank. However generator dynamic response is used to provide reactive reserves in case of contingency.
2. The distribution system must have enough reactive power support so that distribution feeders have a net unity power factor or even slightly leading power factor at low loads and so that transmission system steady state reactive power is essentially all provided by transmission system capacitors.

1.3. Reactive power management in presence of distributed generation

Electric power generation that is integrated within the distribution systems are known as "distributed" or "dispersed" generation (DG). The DG source or technology can be a traditional combustion generator (such as diesel reciprocating generator and natural gas-turbine) and non-traditional generator including fuel cell, storage device and renewable energy source (such as wind turbine and photovoltaic). Non-utility companies are investing in distribution system to meet the active power demand (MW) and get the maximum profit. For example in United States the percentage of nonutility generators (NUGs) has increased from 40GW to more than 150 GW in 10 years from 1990 to 2000 [28–32].

DGs are mainly considered as active source of energy [33], however at higher system loading with maximum DG penetration, the poor voltage profile can be a big challenge for the system operator thus the reactive power compensation approach must be utilized to maintain the voltages in allowable limits [34]. In restructured power system, the system operator is responsible to maintain the allowable voltages by call upon the GENCOs to produce more or less reactive power, by adjusting the field current, by adding or removing reactive power devices (capacitor and reactors) or changing the tap changer position on the transformer. The author in [35] has analyzed the importance of reactive power in presence of DG and conclude that the presence of DG may results in voltage rise problem at light load, thus the voltage regulating device must be also presented. The energy curtailment from DG is not a good solution as this will result in revenue lost. DG presence in the system also affects the reactive power management plan. For example in case of wind generation, asynchronous induction generators are used. Such generators need reactive power from the system to which they are connected. Different methods of reactive compensations are stated in literature [36] including synchronous generator, shunt capacitor banks and end-user reactive power compensation within the reactive power consumption equipment. Here it is also need to be mentioned that the growing trend of using non-conventional power generation (using wind and solar energy) has led to the bounding that the renewable energy generation must also play their role in improving the voltage profile and providing necessary reactive power support. Now-a-days state of the art technology has come out, the wind generation is now using doubly fed induction generator and PV inverters are using special self-commutated line inverter, capable of absorbing and supplying reactive power at different system loading. The reactive power capability of solar and wind power plants can be further enhanced by the addition of SVC, STATCOMS and other reactive support equipment at the plant level. Currently, inverter-based reactive capability is more costly compared to the same capability supplied by synchronous machines [37–39].

Thus the importance of reactive power compensation has been significantly increased due to restructuring of power market and conversion of passive network to active network. Utilities are also focussing towards reduction in power losses and utilizing the distribution lines efficiently. SCB being the low initial cost, no maintenance and no personnel cost is the most cost effective solution for reactive power compensation. Section 2 will present a detailed overview of capacitor bank placement techniques stated in the literature. The non-optimum placement or sizing of SCB may result in increased power losses as losses versus capacitive MVAr follows the deep bath curve relationship [40]. Thus it is necessary to optimally place VAR equipment in the distribution system.

2. Optimum shunt capacitor placement techniques—A review

In literature, different authors have proposed different methods considering different fitness function including minimization of power losses, reduction in installation cost, improvement in voltage profile, lessen the burden on existing lines, maximization of system stability and others. Shunt capacitors are placed in a combination of fixed and switched (variable) capacitor banks. The size of fixed capacitor bank depends on average reactive power demand of the system while switched capacitor banks supply difference of current reactive power demand and fixed capacitive power available. Special control mechanism is used to control the switched capacitor bank power. Shunt capacitors (Q_C) are available in market in discrete sizes which are multiple (k) of smallest capacitor size ($Q_{\min}$), given by Eq. (2) [41,42]. However authors have proposed such methods which results in both continuous and discrete capacitor size. In case of continuous capacitor size, it will be assumed that the capacitor size will be a combination of fixed and switched capacitor. The coarse tuning of capacitor under steady state condition will be achieved using switched capacitor bank.

$$Q_C = k \times Q_{\min} \tag{2}$$

The capacitor placement problems from the literature can be categorized into analytical methods, numerical programming methods, heuristic methods and artificial intelligent methods. Authors have also combined the capacitor problem with other power system problems including distributed generation, network reconfiguration and voltage regulator.

2.1. Analytical methods

In earlier time, when powerful computing resources were unavailable or expensive, the author proposed calculus based analytical algorithm. The authors have also made some approximation in order to reduce the computation procedure. Analytical methods have also been proposed in optimum capacitor placement and sizing. The initial work was carried out by Neagle [1] in 1956 for optimum single and multiple capacitor bank in case of uniform and non-uniform distribution of load. He suggested that in uniformly distributed load the capacitor bank must be placed at $1-(1/2)$ (capacitive kvar/system kvar) distance from the main substation. Cook worked on the same guideline of [1] and proposed a more practical algorithm for fixed capacitor bank for uniformly distributed considering average reactive load in the system [43]. Cook [43] suggested that the optimum location of SCB must be $2/3$ (Reactive load factor). Cook extended his work in [44] to include switched capacitors. Later on several other analytical based methods have been proposed for the capacitor placement [45,46]. Schmill [47] extended the work of Cook [43]. Equations are

given for sizing and placement of n capacitors on a uniform feeder with a uniformly distributed load. The necessary conditions for optimal sizing and placement of one or two capacitors on a feeder with discrete loads and non-uniform resistance are presented. An iterative approach is suggested to solve the problem. Chang et al. [48,49] assumes a feeder with a uniform load and a concentrated end load. Accounting for both peak power losses and energy losses, he determines the optimal location of a fixed capacitor and the resulting savings, given the capacitor size. The optimal solution is determined by considering each of the available capacitor sizes.

Analytical methods are considered as simple methods, however implication of such methods to solve the capacitor placement problem required all assumptions (e.g. load variation neglected) and scenarios (e.g. distributed non-distributed) as considered by the author in developing that algorithm. The author in [50] has indicated that the famous "2/3 rule" may results in negative saving if considered in different scenario. Here it is also noted that most of the analytical methods discussed earlier considered modelling of the capacitor placement locations and sizes as continuous variables. Therefore, the results would need to be rounded up or down to the nearest practical value, which may result in an overvoltage problem or loss savings ($) less than the calculated one. The more recent analytical methods [46,51–54] are much more accurate and practical for distribution systems.

2.2. Numerical programming methods

Numerical Programming is a technique by which mathematical problems are formulated so that they can be solved with arithmetic operations. Numerical programming methods are iterative techniques used to maximize (or minimize) an objective function of decision variables. The values of the decision variables must also satisfy a set of constraints. With the availability of fast computing skills and large memory availability, the utilization of numerical methods in power system has increased [55,56]. In optimum capacitor location problem, authors have formulated different mathematical models and have utilized numerical methods to solve the problem. Duran in [57], have used the dynamic programming approach and implemented the Schmil work [47] of uniformly and randomly distributed load to find the optimum capacitor placement. The formulation in [57] is simple and only considers the energy loss reduction and accounts for discrete capacitor sizes. Fawzi et al. [58] extended the work of [57] and included the released kV A into the savings function. Ponnavsikko and Rao [59] used a numerical method called the method of local variations and further expanded the problem to include the effects of load growth, and switched capacitors for varying load. Lee [50] has proposed iterative based optimization technique considering net monetary saving as convergence criteria in optimum placement of fixed and switched capacitor banks. Later on Baran and Wu [60,61] used the mixed integer programing approach to solve the capacitor problem. Sharaf used the full load flow model to find the optimum shunt capacitor placement in distribution system. He mentioned that the equivalent model proposed in [62] is not useful in finding optimum SCB location as the receiving-end voltage on a distribution system decreases quadratically as system load increases [63]. Khodr [64] solved the SCB problem using mixed-integer linear problem and considered overall energy saving as fitness function.

2.3. Heuristics methods

Heuristics methods are "hints", "suggestions", or "rules of thumb" based methods that are developed through intuition, experience, and judgment. Heuristic methods produce fast and practical strategies which reduce the exhaustive search space and can lead to a solution that is nearer to the optimal solution with confidence [65,66]. Heuristic based techniques are also commonly used to solve shunt capacitor placement problem [3,67–71]. Abdel-Salam et al. [67] proposed a heuristic technique based on identifying the sensitive node and placing such capacitor bank (size) giving the greatest loss reduction due to capacitor placement. Chis et al. in [68] extended the work of [67] and considered cost of capacitor bank (size) from both energy and peak power loss reductions. In [69], the author has evaluated bus-bar sensitivity index to decide the capacitor position(s), sigmoid function is utilized to find the discrete value of capacitor size. In latest research [70], the author has used node voltage stability index to find the candidate bus for capacitor position and maximization of net savings from power loss reduction and the capacitors investment as optimum capacitor size. In [40], the author has considered the end of the weakest line as the potential candidate for optimum shunt capacitor placement and capacitor size is selected corresponding to the minimum power losses using PSO algorithm. In [71], the author has used direct search algorithm to determine the optimal sizes of fixed and switched capacitors together with their optimal locations in a radial distribution system so that net savings are maximized and improvement in the voltage profile is achieved.

2.4. Artificial intelligent methods

The simplest of search algorithms in optimization is exhaustive search that tries all possible solutions from a predetermined set and subsequently picks the best one. However such methods are considered as non-efficient in terms of computation time and space requirement. Intelligent, greedy and nature observed heuristic techniques have also been proposed in literature [72], commonly known as artificial intelligent (AI) methods. AI methods are a special class of heuristic search methods [72]. Intelligent based optimization methods have also been utilized in finding optimum shunt capacitor location and sizing. Authors have used genetic algorithms (GA) [3,73–77] , fuzzy [78,79], immune algorithm [80,81], Tabu search [82], fuzzy-GA [83], particle swarm optimization [84], plant growth simulation algorithm [41,85], memetic-algorithm approach [86], teaching learning based optimization algorithm [87], ant colony [88], graph search algorithm [89] and hybrid algorithm [90–92].

In [93], the author has used non-dominated sorting genetic algorithm (NSGA) to solve multi-criteria problem in capacitor placement. A multi-criteria optimization problem requires simultaneous optimization of a number of objectives with different individual optima [11,14]. Objectives are such that none of them can be improved without degradation of another. Hence, instead of a unique optimal solution, there exists a set of optimal trade-offs between the objectives, the so-called pareto-optimal solutions.

The author in [94,95] has presented an algorithm for optimizing shunt capacitor sizes on radial distribution lines with non-sinusoidal substation voltages, such that the rms voltages and their corresponding total harmonic distortion lie within prescribed values. The result shows that the optimal capacitor sizes found by neglecting the harmonic components may result in unacceptable voltage distortion levels. In [96], the author has considered the presence of non-linear load in distribution system in solving the optimal capacitor placement problem. The authors has used PSO to search for optimal locations, types, and sizes of capacitors to be placed and optimal numbers of switched capacitor banks at different load levels. In [97], the author has solved the problem of capacitor placement using genetic algorithm (GA) in the presence of voltage and current harmonics. The author has claimed that the proposed method results in lower THD_V and greater annual benefits as compared to [98–100]. Presently AI methods are

Table 1
Criterion for comparing the different methods of SCB Placement.

Parameters	Formulae	Abbreviations
Total Line Loss Reduction (TLLR)	$TLLR\% = \frac{\mathrm{Re}\{losses\}_0 - \mathrm{Re}\{losses\}_{SCB}}{\mathrm{Re}\{losses\}_0} \times 100$	▪ Subscript (0) is representing the base case when no SCB is present in the system.
Voltage Profile Improvement (VPI)	$VPI\% = \frac{(\sum_{i=1}^{nbus} V_i L_i)_0 - (\sum_{i=1}^{nbus} V_i L_i)_{SCB}}{(\sum_{i=1}^{nbus} V_i L_i)_0} \times 100$	▪ Subscript (SCB) is representing the case when SCB is present in the system. ▪ λ_{max} is maximum loadability of the system.
System Maximum Loading Improvement (SMLI)	$SMLI\% = \frac{\lambda_{\max(SCB)} - \lambda_{\max(0)}}{\lambda_{\max(0)}} \times 100$	▪ V_i is voltage magnitude at bus i. ▪ *Li* is active load at bus i pu. ▪ *nbus* is total number of buses.
Line Limit	$I_k^{SCB} > I_k^0$ where $1 \le k \le nbr$	▪ I_k is branch current in k_{th} branch. ▪ *nbr* is total number of branches.

considered as the most powerful methods in solving many power system problems including single-objective-multi-constraints, multi-objective-multi-constraints or multi-criteria-pareto-optimal problems. However in large system, AI methods suffers from high computation time and large memory space requirement, in such cases hybrid methods are considered more powerful.

2.5. Multi-dimensional problems

In multi-dimensional problems, the authors have combined the capacitor placement problems with other power system problems including network reconfiguration [101–107], DG placement [108–111], voltage regulators [112–118] and load tap changer [119]. The author in [112–114] considered the capacitor placement problem a multi-dimensional problem and incorporated the voltage regulator placement in addition to control the volt/var. The author in [120] has combined different type of DGs, shunt capacitor and network reconfiguration in a single problem and stated that power losses are significantly reduced when all three objectives are solved simultaneously.

In next section (Section 3), six different methods based on minimization of power losses, weakest voltage bus and maximization of system loadability for optimum shunt capacitor placement will be presented and discussed in detail.

3. Comparitive study

In this section, six different methods for optimum shunt capacitor placement and sizing are compared on the basis of power losses reduction, voltage profile improvement, maximization of system loadability and line limit constraint, summarized in Table 1.

Following constraint, given by Eqs. (3) and (4) is used throughout the analysis:

$$Position\ of\ SCB:\ 2 \le Q_{SCB}\ Position \le nbus \tag{3}$$

$$Size\ of\ SCB:\ Q_{SCB} \le \sum_{i=2}^{nbus} Q_i \tag{4}$$

where *nbus* is total number of buses and Q_i is the reactive power load at bus *i*.

The brief overview of methods is discussed below:

Method 1: Analytical method [121]
In analytical approach, a loss sensitivity factor ($\partial P_{lossj}/\partial P_i$) is formulated for the determination of the optimum size and location of capacitor bank to minimize total power losses. Like other methods, analytical method does not use admittance matrix, inverse of admittance matrix or Jacobian matrix. This method is based on the equivalent current injection that uses the bus-injection to branch-current (BIBC) and branch-current to bus-voltage (BCBV) matrices.

Method 2: Grid search algorithm [122]
In grid search algorithm, shunt capacitor is added to each bus and the size of capacitor is changed from 0% to 100% of total connected load in small steps. Minimization of losses is considered as the objective function. For this purpose, successive load flow methods are used for each step of capacitor size. The main constraints are to restrain the maximum capacitor size selected as total load size.

Method 3: Golden section search algorithm [122]
In golden section search algorithm, the searching space is decreased by checking some discrete values of capacitor size only, in every iteration. Minimization of losses is considered as an objective function. The main constraints are to restrain the maximum capacitor size selected as total load size.
Methods 1–3 is implemented using the MATLAB based voltage stability and optimization programming (VS&OP) tool [122].

Method 4: Optimum shunt capacitor placement and sizing based on min of losses using PSO
In this method, the optimum location and optimum size is determined simultaneously using particle swarm optimization (PSO) algorithm. PSO has advantage over other methods in terms of simplicity, fast convergence as compared to other algorithms (e.g. genetic algorithm) [123,124]. The brief overview of PSO method is presented in the appendix. Minimization of active I^2R power losses (P_L), given by Eq. (5) is considered as fitness function.

$$f = Min\left\{P_L = \sum_{i=1}^{nbr} |I_i|^2 R_i\right\} \tag{5}$$

where *I* is the line current, *R* is the resistance of branch *i*. *nbr* is total number of branches.

Method 5: Hybrid method based on weakest line and minimization of power system losses using particle swarm optimization
In this method, the *SI index* proposed in [125] is utilized to find the weakest voltage bus in power system which could lead to voltage instability, when the load will cross the critical limit. The value of index is given by Eq. (6) and termed as stability index (SI).

$$SI(r) = |V_s|^4 - 4 \times \{P_r x_{ij} - Q_s r_{ij}\}^2 - 4 \times \{P_r r_{ij} + Q_r x_{ij}\}^2 \times |V_s|^2 \ge 0 \tag{6}$$

where V_s is sending end voltage, P_r is active load at receiving end, Q_r is reactive load at receiving end, r_{ij} is resistance of the line *i–j*, *x (ij)* is reactance of the line *i–j*.
Under stable operation, the value of SI should be greater than zero for all buses. When the value of SI becomes closer to one, all buses become more stable. In this algorithm, SI value is calculated for each bus in the network and sort from highest to lowest value. For the bus having the lowest value of SI, the SCB will be placed at that bus. Once the SCB location has been identified, the size of capacitor bank is calculated based on minimization of active I^2R

Table 2
Base case (before capacitor placement).

Test system	P-loss (MW)	System loadability	V_{max}/ V_{min}	Sum (V_iL_i)	Weakest voltage bus
12-Bus	0.0207	5.32	1/0.9439	0.4204	12
30-Bus	0.8819	2.79	1/0.8825	8.0290	27
33-Bus	0.2110	3.41	1/0.9038	3.5228	18
69-Bus	0.2250	3.22	1/0.9092	3.6191	65

Table 3
Capacitor placement based on different methods.

Method	Capacitor position/ size (MVAr)	P-loss (MW)	System loadability	V_{max}/ V_{min}	Sum (V_iL_i)
Method 1: Analytical method					
12-Bus	9/0.2103	0.0126	5.65	1/0.9563	0.4236
30-Bus	22/3.13572	0.6831	2.94	1/0.8979	8.1141
33-Bus	30/1.2298	0.1514	3.6	1/0.9162	3.5651
69-Bus	61/1.2920	0.1521	3.49	1/0.9302	3.6582
Method 2: Grid search algorithm					
12-Bus	9/0.2106	0.0126	5.65	1/0.9563	0.4236
30-Bus	21/3.45818	0.6812	2.93	1/0.8979	8.1183
33-Bus	30/1.265	0.1514	3.6	1/0.9165	3.5662
69-Bus	61/1.3203	0.1520	3.49	1/0.9306	3.6590
Method 3: Golden section search algorithm					
12-Bus	9/0.2102	0.0126	5.65	1/0.9563	0.4236
30-Bus	21/3.4347	0.6812	2.93	1/0.8978	8.1178
33-Bus	30/1.258	0.1514	3.6	1/0.9165	3.5660
69-Bus	61/1.330	0.1520	3.49	1/0.9307	3.6592
Method 4: Minimization of power losses					
12-Bus	9/0.2102	0.0126	5.65	1/0.9563	0.4236
30-Bus	21/3.4347	0.6812	2.93	1/0.8978	8.1178
33-Bus	30/1.2580	0.1514	3.6	1/0.9165	3.5660
69-Bus	61/1.330	0.1520	3.49	1/0.9307	3.6592
Method 5: Hybrid (voltage stability + power losses)					
12-Bus	12/0.1737	0.0134	5.66	1/0.9561	0.4232
30-Bus	27/2.384	0.7148	2.95	1/0.8971	8.0956
33-Bus	18/0.4798	0.1891	3.56	1/0.9211	3.5471
69-Bus	65/0.9822	0.1691	3.42	1/0.9280	3.6496
Method 6: Maximization of system loadability					
12-Bus	11/0.4300	0.0263	6.06	1/0.9710	0.4266
30-Bus	24/5.15	0.8647	3.06	1/0.9086	8.1562
33-Bus	12/2.260	0.2733	3.81	1/0.9357	3.6069
69-Bus	62/2.64	0.2214	3.72	1/0.9493	3.6923

power losses (P_L), given by Eq. (5) using PSO optimization algorithm. The similar approach, based on weaest voltage bus has been utilized in [40] and [70].

Method 6: Optimum shunt capacitor placement and sizing based on maximization of system loadability using PSO (proposed method)
In this method, PSO method will be utilized to find the optimum SCB position and optimum size, based on maximization of system loadability (λ_{max}) as given by Eq. (7).

$$f = Max\{\lambda_{\max}\} \quad (7)$$

To find the maximum loadability of the system ($\lambda_{\max}$), the active and reactive load is increased on all buses (using Eqs. 8a and 8b) with equal loading factor of 0.05, till the divergence is observed in load flow analysis. The author in [126] has used this method for optimum network reconfiguration in distribution system.

$$P_{new} = P_0 \times \text{loading factor}(\lambda) \quad (8a)$$

$$Q_{new} = Q_0 \times \text{loading factor}(\lambda) \quad (8b)$$

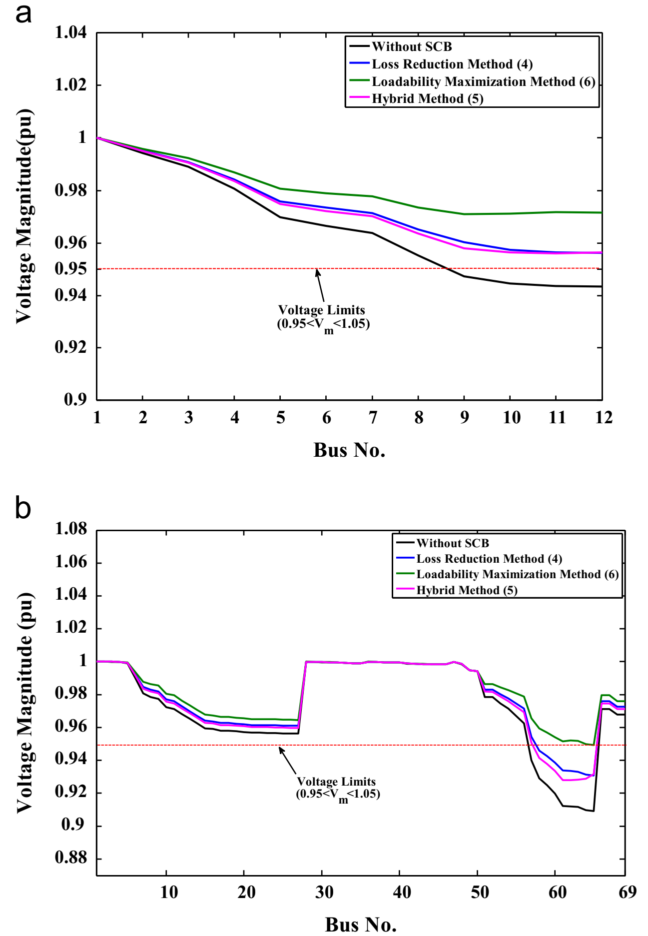

Fig. 2. Comparison of voltage profile using different methods of shunt capacitor placement. (a) 12-Bus Test System, and (b) 69-Bus Test System.

where λ is a loading factor. P_o and Q_o is initial active and reactive power load, connected with *i*th bus. P_{new} and Q_{new} is final active and reactive power load, connected with *i*th bus.

3.1. Simulation and results

The proposed method is tested on 12-bus [127], 30-bus [128], 33-bus [129] and 69-bus [60] radial distribution test systems in all three scenarios. Thukaram load flow method [130] is used to carry out the power flow analysis for the radial distribution system. In base case, when no capacitor is place in the system, following results have been obtained, given in Table 2.

Table 3 is showing the results, when different methods of optimum capacitor placement and sizing are applied on test systems.

4. Discussion

The above methods are compared on the basis of different criterion given in Table 1. Here it is interesting to note that the method 1 to method 4, approximately give the same results. Thus these methods will be discussed simultaneously as “loss reduction methods”. Following important points are concluded:

1. There are significant improvements in reduction in I^2R power losses, voltage profile improvement and in maximization of system loadability ($\lambda_{\max}$) in comparison with base case (no capacitor). However the method based on loadability

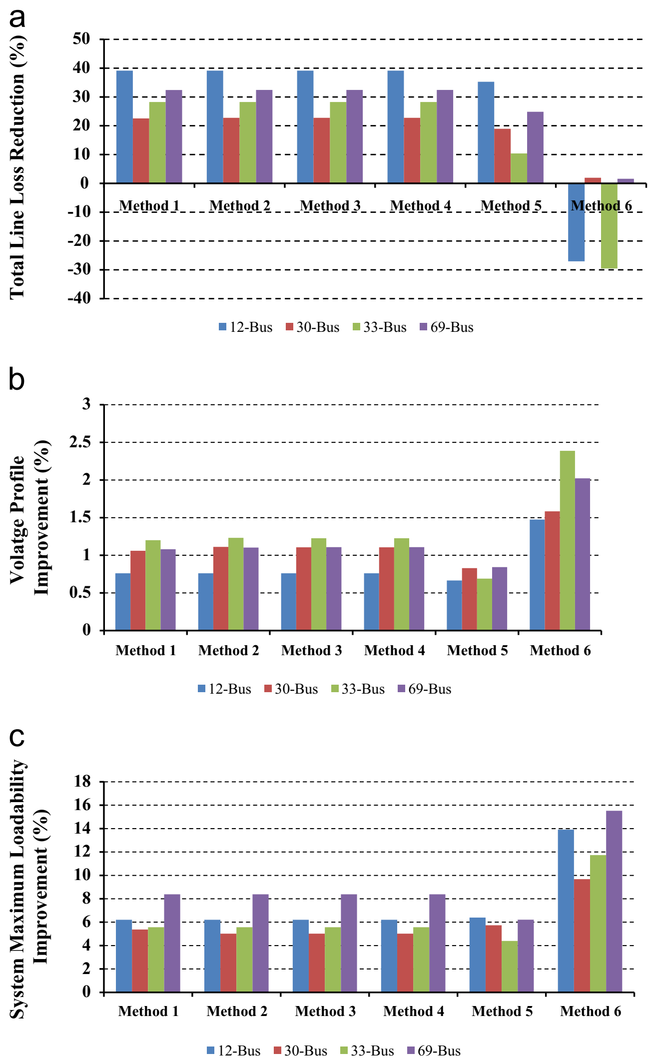

Fig. 3. Comparison of six different methods of optimum shunt capacitor placement. (a) Total Line Loss reduction, (b) Voltage Profile Improvement, and (c) System Maximum Loadability Improvement.

Table 4
Maximum loadability improvement due to single capacitor placement considering voltage limits.

Methods	Capacitor position	Capacitor size (MVAR)	Maximum loading margin	Maximum load (kV A)
Base case	–	–	0.89	529.91
Analytical	9	0.2103	1.11	654.94
Grid search	9	0.2106	1.11	654.94
Golden section Search	9	0.2102	1.11	654.94
Hybrid	12	0.1737	1.11	654.94
Losses	9	0.2102	1.11	654.94
Loadability	11	0.4300	1.39	827.61

maximization (method 6) gives higher losses in some cases (12-bus and 33-bus test systems).

2. Here it is also need to be noted that the capacitor size in case of loadability maximization (method 6) have also been significantly increased as compared to other methods. However in case of capacitor placement based on weakest voltage bus (method 5) is much lesser than any other method.
3. The voltage profile improvement in case of loadability maximization (method 6) was found better than the other methods in all test cases, as shown in Fig. 2 (only two bus test system are presented in the result).
4 To visualize the impact of capacitor placement on power loss reduction, voltage profile improvement and maximum loadability improvement, the formulae defined in Table 1 is calculated and plotted in Fig. 3.
From Fig. 3, it can be seen that the losses have been reduced in case of methods 1-5, however the method 6 can increase the system losses. However in terms of VPI and maximum loadability improvement, method 6 was found better than the other methods.
5 To observe the maximum amount of load (kV A) the system can sustain, the load on each bus in 12-bus test system in presence of capacitor is increased till the allowable voltage limit is reached ($0.95 < Vbus < 1.05$) and the results are summarized in Table 4. Here it can be seen that using the method of loadability maximization, the system can sustain 1.39 times of base load (i.e. 1.39×595.4 kV A = 827.606 kV A), however other methods can sustain only 1.1 times of base load (i.e. 1.1×595.4 kV A = 654.94 kV A).
6 One of the important factors that are needed to be considered in capacitor placement is reactive line current. The placement of capacitor may increase the reactive loading on some of the lines, as shown in Fig. 4. It is interesting to note that the reactive line loading on some of the lines have been increased in case of weakest voltage bus approach (method 5) and maximization of system loadability (method 6) as compared to base case and minimization of loss approach. The amount of reactive current can be reduced by placement of multi-capacitor units in the system.

From the above discussion it can be concluded that the capacitor placement and sizing is based on different parameters including minimization of power losses, voltage profile improvement, maximization of system capacity and line limit constraint. Another major factor that is needed to be considered is capital cost, the capital cost will include capacitor cost, protection system used and cost of constructing new lines (if needed). Methods 1–4 of shunt capacitor placement in the system can well handle the minimization of power losses problem considering the line limit constraints, with additional benefits of maximization of system loading and voltage profile improvement.

5. Conclusion

This paper has presented a very detailed overview of optimum shunt capacitor bank (SCB) placement in distribution system. In literature, analytical, numerical programming, heuristic and artificial intelligent based techniques have been proposed. The paper has also presented a comparative study of six different techniques of optimum capacitor placement based on minimization of power losses, weakest voltage bus and maximization of system loadability. The results show that the optimum capacitor placement based on minimization of power losses helps in reducing the reactive current component in total I^2R losses, in addition to fulfill the line current constraint. The other advantages including voltage profile improvement and maximization of system loadability are also achieved on small scale using power loss reduction technique. Hybrid method (weakest voltage bus and minimization of power losses) and maximization of system loadability must be utilized carefully, such approaches may results in increasing the

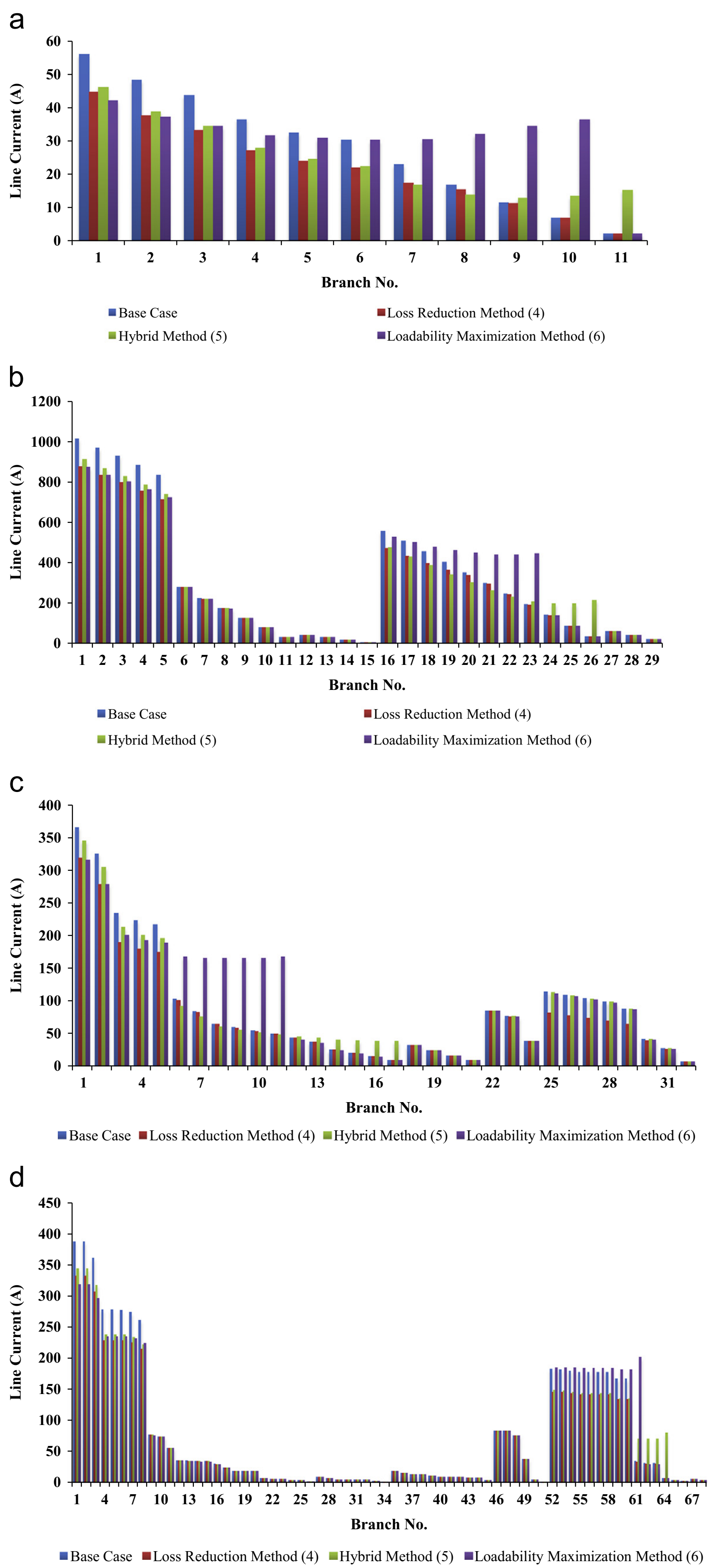

Fig. 4. Comparison of line current due to loss reduction method, hybrid method and maximization of system loadability. (a) 12-Bus Test System, (b) 30-Bus Test System, (c) 33-Bus Test System, and (d) 69-Bus Test System.

Table A.1
Proposed Algorithm for Single-DG Units Placement

Steps	Disscussion
Initialization	• Initialize PSO parameters. • A particle *i* among initial populations (d) is a one or two dimension position vector x_i, representing shunt capacitor bank position (Q_{SCB}) and shunt capacitor sizes ($\mathbf{Q}_{SCB}$) or both. $x_i=[Q_{SCB}, \mathbf{Q}_{SCB}]$
Generate Initial Solutions	Calculate the best fitness function among initial solution of x_i, considering the constraint given in *Eqns. (3 and 4)*.
Update Initial p_{best} and g_{best}	For Iteration = 1, find initial p_{best} and g_{best}.
Generate new set of positions	• Update velocity and position vectors using *Eqns. (A-1) and (A-2)*. • Check the feasibility of the new generated particles xi, given in *Eqns. (3 and 4)*.
Update Initial p_{best} and g_{best}	• Calculate new fitness function of updated particles. • Update p_{best} and g_{best}.
Stopping Criteria	Update Iteration If Iteration = = Iteration$_{(max)}$ End

reactive line current in some of the lines if constraints are not defined properly.

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Appendix

Particle swarm optimization (PSO) was introduced by [131] to solve the optimization problem, based on social–psychological metaphor behavior. A particle *i* is represented as $x_i=(x_{i1}, x_{i2}, x_{i3}, x_{i4}, \ldots\ldots, x_{id})$. The position associated with the best fitness $pbest_i=(pbest_{i1}, pbest_{i2}, pbest_{i3}, pbest_{i4} \ldots\ldots\ldots pbest_{id})$ is considered as its current best position. Here *d* is a total number of initial particles. The overall best position of the population associated with the current overall best fitness value g_{best} is recorded. The rate of the position of *i*th particle is represented as $v_i=(v_{i1}, v_{i2}, v_{i3}, v_{i4}, \ldots\ldots v_{id})$. During the iteration procedure, the velocity and the position of *i*th particles are updated according to the Eqs. (A-1) and (A-2):

$$v_{id}^{(t+1)}=w\times v_{id}^{(t)}+c_1\times r_1\times(pbest_{id}-x_{id})+c_2\times r_2\times(gbest-x_{id}) \quad (A-1)$$

$$x_{id}^{(t+1)}=x_{id}^{(t)}+v_{id}^{(t+1)} \quad (A-2)$$

where *t* is number of iterations, c_1 and c_2 are constants (0.7), r_1 and r_2 are random numbers, *w* is inertia weight given by Eq. (A-3).

$$w=w_{\max}+\frac{w_{\max}-w_{\min}}{iter_{\max}}\times t \quad (A-3)$$

where $w_{\max}$ and $w_{\min}$ is 0.9 and 0.4 respectively, c_1 and c_2 are constants ($c_1=c_2=0.7$).

Table A1 Shows the complete steps of the proposed algorithm for optimum single unit capacitor placement and sizing.

In this paper, PSO is used to achieve the desired fitness function (minimization of power losses or maximization of system loadability). In the present case of shunt placement and sizing, the *i*th particle (x_i) is a two dimension vector (Q_{SCB}, $\mathbf{Q}_{SCB}$), representing random shunt capacitor positions (Q_{SCB}) and DG sizes ($\mathbf{Q}_{SCB}$). The algorithm can also be used for multi SCB placement, considering (x_i) is a six dimension vector (Q_{SCB1}, $\mathbf{Q}_{SCB1}$, Q_{SCB2}, $\mathbf{Q}_{SCB2}$, Q_{SCB3}, $\mathbf{Q}_{SCB3}$) representing three SCB position (Q_{SCB}) and three SCB size ($\mathbf{Q}_{SCB}$).

References

[1] Neagle NM, Samson DR. Loss reduction from capacitors installed on primary feeders. Power Apparatus Syst Part III Trans Am Inst Electr Eng 1956;75:950–9.
[2] Kasztenny B, Schaefer J, Clark E. Fundamentals of adaptive protection of large capacitor banks. Power Systems Conference: Advanced Metering, Protection, Control, Communication, and Distributed Resources; 2007 PSC 2007: IEEE; 2007. p. 154–86.
[3] Segura S, Romero R, Rider MJ. Efficient heuristic algorithm used for optimal capacitor placement in distribution systems. Int J Electr Power Energy Syst 2010;32:71–8.
[4] Lidula NWA, Rajapakse AD. Microgrids research: a review of experimental microgrids and test systems. Renewable Sustainable Energy Rev 2011;15:186–202.
[5] Eltawil MA, Zhao Z. Grid-connected photovoltaic power systems: technical and potential problems—a review. Renewable Sustainable Energy Rev 2010;14:112–29.
[6] Taylor CW. Shunt compensation for voltage stability. Power plants and power systems control 2003. In: A proceedings volume from the fifth IFAC symposium, Seoul, South Korea; 15–19 September 2003: Gulf Professional Publishing; 2004. p. 43.
[7] Andersson G, Donalek P, Farmer R, Hatziargyriou N, Kamwa I, Kundur P, et al. Causes of the 2003 major grid blackouts in North America and Europe, and recommended means to improve system dynamic performance. IEEE Trans Power Syst 2005;20:1922–8.
[8] Pereira L. Cascade to black [system blackouts]. Power Energy Mag, IEEE 2004;2:54–7.
[9] Burkhart LA. FERC takes on reactive power. Public utilities fortnightly. spark, letter #15. March 2005 Available from 〈http://www.pur.com/pubs/spark/mar05.pdfSimilar〉 Last Accessed on 17-Sep-2013. 2005.
[10] Taylor C, Van Leuven A. CAPS: improving power system stability using the time-overvoltage capability of large shunt capacitor banks. IEEE Trans Power Delivery 1996;11:783–92.
[11] Bruns DP, Newcomb GR, Miske Jr SA, Taylor CW, Lee GE, Edris A. Shunt capacitor bank series group shorting (CAPS) design and application. IEEE Trans Power Delivery 2001;16:24–32.
[12] Dortolina CA, Nadira R. The loss that is unknown is no loss at all: a top-down/bottom-up approach for estimating distribution losses. IEEE Trans Power Syst 2005;20:1119–25.
[13] Targosz R, Belmans R, Declercq J, De Keulenaer H, Furuya K, Karmarkar M, et al. The potential for global energy savings from high efficiency distribution transformers. Leonardo Energy Transformer–European Copper Institute; 2005.
[14] EPRI. Assessment of transmission and distribution losses in New York. Available from 〈http://www.nyserda.ny.gov/Publications/Research-and-Development-Technical-Reports/-/media/Files/Publications/Research/Electic%

20Power%20Delivery/2012-11-12_epri_assessment_losses_report.pdf⟩. Last accessed on 17-Sep-2013. November 2012.
[15] EIA. Annual Energy Outlook 2011. Available from ⟨http://www.eia.gov/forecasts/archive/aeo11/⟩ Last Access on: 17-Sep-2013. Energy Information Administration, Washington, DC. 2011.
[16] Olivares JC, Liu Y, Cañedo JM, Escarela-Pérez R, Driesen J, Moreno P. Reducing losses in distribution transformers. IEEE Trans Power Delivery 2003;18:821–6.
[17] Prada RB, Souza LJ. Voltage stability and thermal limit: constraints on the maximum loading of electrical energy distribution feeders. IEE Proc Gener, Transm Distrib 1998;145:573–7.
[18] ENERGI. Guidelines to install, operate and maintain ht capacitors & it's associated equipment ⟨http://www.energegroup.com/CapacitorManual.pdf⟩.
[19] Barry D. Increasing renewable energy accessibility in Ireland. In: Proceedings of the 19th World Energy Congress. 2004:1-10.
[20] Aman MM, Jasmon GB, Mokhlis H, Bakar AHA. Analysis of the performance of domestic lighting lamps. Energy Policy 2013;52:482–500.
[21] Milligan M, Ela E, Hein J, Schneider T, Brinkman G, Denholm P. Volume 4: Bulk electric power systems: operations and transmission planning (2012). Exploration of high-penetration renewable electricity futures. Vol. 4 of renewable electricity futures Study. NREL/TP-6A20-52409-4. Golden, CO: National Renewable Energy Laboratory. 2012.
[22] EURELECTRIC. Power outages in 2003—global regulatory Network. Available from ⟨http://www.globalregulatorynetwork.org/Resources/PowerOutagesin2003.pdf⟩ Last Accessed on 17-Sep-2013. 2004.
[23] Root CE. The future beckons [electric power industry]. IEEE Power Energy Mag 2006;4:24–31.
[24] Singh H, Hao S, Papalexopoulos A. Transmission congestion management in competitive electricity markets. IEEE Trans Power Syst 1998;13:672–80.
[25] Hemmati R, Hooshmand R-A, Khodabakhshian A. State-of-the-art of transmission expansion planning: comprehensive review. Renewable Sustainable Energy Rev 2013;23:312–9.
[26] Baldick R. Reactive issues-reactive power in restructured markets. IEEE Power Energy Mag 2004;2:14–7.
[27] Hogan WW. Markets in real electric networks require reactive prices. Electricity transmission pricing and technology. Springer; 1996; 143–82.
[28] Jiayi H, Chuanwen J, Rong X. A review on distributed energy resources and MicroGrid. Renewable Sustainable Energy Rev 2008;12:2472–83.
[29] Tan W-S, Hassan MY, Majid MS, Abdul Rahman H. Optimal distributed renewable generation planning: a review of different approaches. Renewable Sustainable Energy Rev 2013;18:626–45.
[30] Moghaddas-Tafreshi SM, Mashhour E. Distributed generation modeling for power flow studies and a three-phase unbalanced power flow solution for radial distribution systems considering distributed generation. Electr Power Syst Res 2009;79:680–6.
[31] Ouyang W, Cheng H, Zhang X, Yao L. Distribution network planning method considering distributed generation for peak cutting. Energy Convers Manage 2010;51:2394–401.
[32] Puttgen HB, Macgregor PR, Lambert FC. Distributed generation: semantic hype or the dawn of a new era? IEEE Power Energy Mag 2003;1:22–9.
[33] Ackermann T, Andersson G, Söder L. Distributed generation: a definition. Electr Power Syst Res 2001;57:195–204.
[34] Mahmud MA, Hossain MJ, Pota HR, Nasiruzzaman ABM. Voltage control of distribution networks with distributed generation using reactive power compensation. In: IECON 2011—37th annual conference on IEEE industrial electronics society 2011. p. 985–90.
[35] Liew SN, Strbac G. Maximising penetration of wind generation in existing distribution networks. IEE Proc Gener, Transm Distrib 2002;149:256–62.
[36] Turitsyn K, Sulc P, Backhaus S, Chertkov M. Options for control of reactive power by distributed photovoltaic generators. Proc IEEE 2011;99:1063–73.
[37] Ellis A, Nelson R, Von Engeln E, Walling R, MacDowell J, Casey L, et al. Reactive power performance requirements for wind and solar plants. Power and energy society general meeting, 2012 IEEE: IEEE; 2012. p. 1–8.
[38] Xu L, Cartwright P. Direct active and reactive power control of DFIG for wind energy generation. IEEE Trans Energy Convers 2006;21:750–8.
[39] Tang Y, Xu L. A flexible active and reactive power control strategy for a variable speed constant frequency generating system. IEEE Trans Power Electr 1995;10:472–8.
[40] Aman MM, Jasmon GB, Bakar AHA, Mokhlis H. Optimum capacitor placement and sizing for distribution system based on an improved voltage stability index. Int Rev Electr Eng 2012;7:4622–30.
[41] Rao RS, Narasimham S, Ramalingaraju M. Optimal capacitor placement in a radial distribution system using plant growth simulation algorithm. Int J Electr Power Energy Syst 2011;33:1133–9.
[42] Mekhamer SF, Soliman SA, Moustafa MA, El-Hawary ME. Load flow solution of radial distribution feeders: a new contribution. Int J Electr Power Energy Syst 2002;24:701–7.
[43] Cook RF. Analysis of capacitor application as affected by load cycle. Power Apparatus Syst Part III Trans Am Inst Electr Eng 1959;78:950–6.
[44] Cook RF. Optimizing the application of shunt capacitors for reactive-volt-Ampere control and loss reduction. Power Apparatus Syst Part III Trans Am Inst Electr Eng 1961;80:430–41.
[45] Bae YG. Analytical method of capacitor allocation on distribution primary feeders. IEEE Trans Power Apparatus Syst. 1978;PAS-97:1232–8.
[46] Grainger JJ, Lee SH. Optimum size and location of shunt capacitors for reduction of losses on distribution feeders. IEEE Trans Power Apparatus Syst. 1981;PAS-100:1105-18.
[47] Schmill JV. Optimum size and location of shunt capacitors on distribution feeders. IEEE Trans Power Apparatus Syst 1965;84:825–32.
[48] Chang NE. Locating shunt capacitors on primary feeder for voltage control and loss reduction. IEEE Trans Power Apparatus Syst 1969:1574–7.
[49] Chang NE. Generalized equations on loss reduction with shunt capacitor. IEEE Trans Power Apparatus Syst 1972:2189–95.
[50] Lee SH, Grainger JJ. Optimum placement of fixed and switched capacitors on primary distribution feeders. IEEE Trans Power Apparatus Syst. 1981;PAS-100:345-52.
[51] Salama M, Chikhani A, Hackam R. Control of reactive power in distribution systems with an end-load and fixed load condition. IEEE Trans Power Apparatus Syst 1985:2779–88.
[52] Salama M, Mansour E, Chikhani A, Hackam R. Control of reactive power in distribution systems with an end-load and varying load condition. IEEE Trans Power Apparatus Syst 1985:941–7.
[53] Haque MH. Capacitor placement in radial distribution systems for loss reduction. IEE Proc Gener Transm Distrib: IEE; 1999. p. 501-5.
[54] Cho M, Chen Y. Fixed/switched type shunt capacitor planning of distribution systems by considering customer load patterns and simplified feeder model. Gener, Transm Distrib, IEE Proc: IET 1997:533–40.
[55] Crow M. Computational methods for electric power systems. Crc Press; 2003.
[56] Saadat H. Power system analysis. Singapore: WCB/McGraw-Hill; 1999.
[57] Duran H. Optimum number, location, and size of shunt capacitors in radial distribution feeders: a dynamic programming approach. IEEE Trans Power Apparatus Syst. 1968; PAS-87:1769-74.
[58] Fawzi TH, El-Sobki SM, Abdel-Halim M. New approach for the application of shunt capacitors to the primary distribution feeders. IEEE Trans Power Apparatus Syst 1983:10–3.
[59] Ponnavsikko M, Rao KP. Optimal choice of fixed and switched shunt capacitors on radial distributors by the method of local variations. IEEE Trans Power Apparatus Syst 1983:1607–15.
[60] Baran M, Wu FF. Optimal sizing of capacitors placed on a radial distribution system. IEEE Trans Power Delivery 1989;4:735–43.
[61] Baran ME, Wu FF. Optimal capacitor placement on radial distribution systems. IEEE Trans Power Delivery 1989;4:725–34.
[62] Jasmon G, Lee L. Distribution network reduction for voltage stability analysis and loadflow calculations. Int J Electr Power Energy Syst 1991;13:9–13.
[63] Sharaf AM, Ibrahim ST. Optimal capacitor placement in distribution networks. Electr Power Syst Res 1996;37:181–7.
[64] Khodr HM, Olsina FG, PMDO-D Jesus, Yusta JM. Maximum savings approach for location and sizing of capacitors in distribution systems. Electr Power Syst Res 2008;78:1192–203.
[65] Minsky ML. Some methods of artificial intelligence and heuristic programming. In: Proceedings of the symposium on the mechanization of thought processes, Teddington1958.
[66] Ng H, Salama M, Chikhani A. Classification of capacitor allocation techniques. IEEE Trans Power Delivery 2000;15:387–92.
[67] Abdel-Salam TS, Chikhani AY, Hackam R. A new technique for loss reduction using compensating capacitors applied to distribution systems with varying load condition. IEEE Trans Power Delivery 1994;9:819–27.
[68] Chis M, Salama M, Jayaram S. Capacitor placement in distribution systems using heuristic search strategies. IEE Proc Gener Transm Distrib 1997;144:225–30.
[69] da Silva IC, Carneiro S, de Oliveira EJ, de Souza Costa J, Rezende Pereira JL, Garcia PAN. A heuristic constructive algorithm for capacitor placement on distribution systems. IEEE Trans Power Syst 2008;23:1619–26.
[70] Hamouda A, Sayah S. Optimal capacitors sizing in distribution feeders using heuristic search based node stability-indices. Int J Electr Power Energy Syst 2013;46:56–64.
[71] Ramalinga Raju M, Ramachandra Murthy K, Ravindra K. Direct search algorithm for capacitive compensation in radial distribution systems. Int J Electr Power Energy Syst 2012;42:24–30.
[72] Kokash N. An introduction to heuristic algorithms. Department of Informatics and Telecommunications 2005.
[73] Sundhararajan S, Pahwa A. Optimal selection of capacitors for radial distribution systems using a genetic algorithm. IEEE Trans Power Syst 1994;9:1499–507.
[74] Das D. Reactive power compensation for radial distribution networks using genetic algorithm. Int J Electr Power Energy Syst 2002;24:573–81.
[75] Haghifam M-R, Malik O. Genetic algorithm-based approach for fixed and switchable capacitors placement in distribution systems with uncertainty and time varying loads. IET Gener Transm Distrib 2007;1:244–52.
[76] Levitin G, Kalyuzhny A, Shenkman A, Chertkov M. Optimal capacitor allocation in distribution systems using a genetic algorithm and a fast energy loss computation technique. IEEE Trans Power Delivery 2000;15:623–8.
[77] Boone G, Chiang H-D. Optimal capacitor placement in distribution systems by genetic algorithm. Int J Electr Power Energy Syst 1993;15:155–61.
[78] Ng H, Salama M, Chikhani A. Capacitor allocation by approximate reasoning: fuzzy capacitor placement. IEEE Trans Power Delivery 2000;15:393–8.
[79] Bhattacharya S, Goswami S. A new fuzzy based solution of the capacitor placement problem in radial distribution system. Expert Syst Appl 2009;36:4207–12.

[80] Shyh-Jier H. An immune-based optimization method to capacitor placement in a radial distribution system. IEEE Trans Power Delivery 2000;15:744–9.
[81] Huang S-J. An immune-based optimization method to capacitor placement in a radial distribution system. IEEE Trans Power Delivery 2000;15:744–9.
[82] Pires DF, Martins AG, Antunes CH. A multiobjective model for VAR planning in radial distribution networks based on tabu search. IEEE Trans Power Syst 2005;20:1089–94.
[83] Das D. Optimal placement of capacitors in radial distribution system using a fuzzy-GA method. Int J Electr Power Energy Syst. 30:361-7.
[84] Singh S, Rao A. Optimal allocation of capacitors in distribution systems using particle swarm optimization. Int J Electr Power Energy Syst 2012; 43:1267–75.
[85] Huang S-J, Liu X-Z. A plant growth-based optimization approach applied to capacitor placement in power systems 2012.
[86] Mendes A, Franca P, Lyra C, Pissarra C, Cavellucci C. Capacitor placement in large-sized radial distribution networks. Gener Transm Distrib, IEE Proc: IET 2005:496–502.
[87] Sultana S, Roy PK. Optimal capacitor placement in radial distribution systems using teaching learning based optimization. Int J Electr Power Energy Syst 2014;54:387–98.
[88] Kaur D, Sharma J. Multiperiod shunt capacitor allocation in radial distribution systems. Int J Electr Power Energy Syst 2013;52:247–53.
[89] Carlisle J, El-Keib A. A graph search algorithm for optimal placement of fixed and switched capacitors on radial distribution systems. IEEE Trans Power Delivery 2000;15:423–8.
[90] Gallego RA, Monticelli AJ, Romero R. Optimal capacitor placement in radial distribution networks. IEEE Trans Power Syst 2001;16:630–7.
[91] Kannan S, Renuga P, Kalyani S, Muthukumaran E. Optimal capacitor placement and sizing using fuzzy-DE and fuzzy-MAPSO methods. Appl Soft Comput 2011;11:4997–5005.
[92] Goswami SK, Ghose T, Basu SK. An approximate method for capacitor placement in distribution system using heuristics and greedy search technique. Electr Power Syst Res 1999;51:143–51.
[93] Milosevic B, Begovic M. Capacitor placement for conservative voltage reduction on distribution feeders. IEEE Trans Power Delivery 2004; 19:1360–7.
[94] Baghzouz Y, Ertem S. Shunt capacitor sizing for radial distribution feeders with distorted substation voltages. IEEE Trans Power Delivery 1990;5:650–7.
[95] Wu Z, Lo K. Optimal choice of fixed and switched capacitors in radial distributors with distorted substation voltage. IEE Proc Gener Transm Distrib 1995;142:24–8.
[96] Yu X-m Xiong X-Y, Wu Y-w. A PSO-based approach to optimal capacitor placement with harmonic distortion consideration. Electr Power Syst Res 2004;71:27–33.
[97] Masoum MAS, Ladjevardi M, Jafarian A, Fuchs EF. Optimal placement, replacement and sizing of capacitor Banks in distorted distribution networks by genetic algorithms. IEEE Trans Power Delivery 2004;19:1794–801.
[98] Masoum M, Ladjevardi M, Fuchs E, Grady E. Optimal placement and sizing of fixed and switched capacitor banks under nonsinusoidal operating conditions. Power Engineering Society Summer Meeting, 2002 IEEE: IEEE; 2002. 807–13.
[99] Masoum MAS, Ladjevardi M, Fuchs EF, Grady WM. Application of local variations and maximum sensitivities selection for optimal placement of shunt capacitor banks under nonsinusoidal operating conditions. Int J Electr Power Energy Syst 2004;26:761–9.
[100] Masoum MA, Jafarian A, Ladjevardi M, Fuchs EF, Grady W. Fuzzy approach for optimal placement and sizing of capacitor banks in the presence of harmonics. IEEE Trans Power Delivery 2004;19:822–9.
[101] Su C-T, Lee C-S. Feeder reconfiguration and capacitor setting for loss reduction of distribution systems. Electr Power Syst Res 2001;58:97–102.
[102] Jiang D, Baldick R. Optimal electric distribution system switch reconfiguration and capacitor control. IEEE Trans Power Syst 1996;11:890–7.
[103] Rezaei P, Vakilian M, Hajipour E. Reconfiguration and capacitor placement in radial distribution systems for loss reduction and reliability enhancement. In: Intelligent system application to power systems (ISAP), 2011 16th international conference on 2011. p. 1–6.
[104] Zeng R, Pan X, He J, Sheng X. Reconfiguration and capacitor placement for loss reduction of distribution system. In: TENCON '02 Proceedings 2002 IEEE Region 10 conference on computers, communications, control and power engineering 2002. p. 1945-9 vol.3.
[105] Chung-Fu C. Reconfiguration and capacitor placement for loss reduction of distribution systems by ant colony search algorithm. IEEE Trans Power Syst 2008;23:1747–55.
[106] Peponis GJ, Papadopoulos MP, Hatziargyriou ND. Distribution network reconfiguration to minimize resistive line losses. IEEE Trans Power Delivery 1995;10:1338–42.
[107] de Oliveira LW, Carneiro Jr S, de Oliveira EJ, Pereira JLR, Silva Jr IC, Costa JS. Optimal reconfiguration and capacitor allocation in radial distribution systems for energy losses minimization. Int J Electr Power Energy Syst 2010;32:840–8.
[108] Sajjadi SM, Haghifam M-R, Salehi J. Simultaneous placement of distributed generation and capacitors in distribution networks considering voltage stability index. Int J Electr Power Energy Syst 2013;46:366–75.
[109] Mahaei SM, Sami T, Shilebaf A, Jafarzadeh J. Simultaneous placement of distributed generations and capacitors with multi-objective function. In: Electrical power distribution networks (EPDC), 2012 proceedings of 17th conference on 2012. p. 1–9.
[110] Mady I. Optimal sizing of capacitor banks and distributed generation in distorted distribution networks by genetic algorithms. In: Electricity distribution—Part 1, 2009 CIRED 2009 20th international conference and exhibition on 2009. p. 1–4.
[111] Moradi MH, Zeinalzadeh A, Mohammadi Y, Abedini M. An efficient hybrid method for solving the optimal sitting and sizing problem of DG and shunt capacitor banks simultaneously based on imperialist competitive algorithm and genetic algorithm. Int J Electr Power Energy Syst 2014;54:101–11.
[112] Grainger J, Civanlar S. Volt/Var control on distribution Systems with lateral branches using shunt capacitors and voltage regulators Part I: The overall problem. IEEE Trans Power Apparatus Syst 1985:3278–83.
[113] Civanlar S, Grainger J. Volt/Var control on distribution systems with lateral branches using shunt capacitors and voltage regulators Part II: The solution method. IEEE Trans Power Apparatus Syst 1985:3284–90.
[114] Civanlar S, Grainger J. Volt/var control on distribution systems with lateral branches using shunt capacitors and voltage regulators Part III: The numerical results. IEEE Trans Power Apparatus Syst 1985:3291–7.
[115] Gu Z, Rizy DT. Neural networks for combined control of capacitor banks and voltage regulators in distribution systems. IEEE Trans Power Delivery 1996;11:1921–8.
[116] Carpinelli G, Noce C, Proto D, Varilone P. Voltage regulators and capacitor placement in three-phase distribution systems with non-linear and unbalanced loads. Int J Emerg Electr Power Syst 2006:7.
[117] Szuvovivski I, Fernandes TSP, Aoki AR. Simultaneous allocation of capacitors and voltage regulators at distribution networks using genetic algorithms and optimal power flow. Int J Electr Power Energy Syst 2012;40:62–9.
[118] Franco JF, Rider MJ, Lavorato M, Romero R. A mixed-integer lp model for the optimal allocation of voltage regulators and capacitors in radial distribution systems. Int J Electr Power Energy Systems 2013;48:123–30.
[119] Ziari I, Ledwich G, Ghosh A. A new technique for optimal allocation and sizing of capacitors and setting of LTC. Int J Electr Power Energy Syst 2013;46:250–7.
[120] Hung DQ, Mithulananthan N, Bansal RA. Combined practical approach for distribution system loss reduction. Int J Ambient Energy 2013:1–22.
[121] Gözel T, Hocaoglu MH. An analytical method for the sizing and siting of distributed generators in radial systems. Electr Power Syst Res 2009; 79:912–8.
[122] Gözel T, Eminoglu U, Hocaoglu MH. A tool for voltage stability and optimization (VS&OP) in radial distribution systems using matlab graphical user interface (GUI). Simul Modell Pract Theor 2008;16:505–18.
[123] Bai Q. Analysis of particle swarm optimization algorithm. Comput Inf Sci 2010;3:P180.
[124] Engelbrecht A. Particle swarm optimization: pitfalls and convergence aspects. Available from ⟨http://www.cs.up.ac.za/cs/engel/PSOtutorialCEC05.pdf⟩ Last accessed on 17-Sep-2013.
[125] Chakravorty M, Das D. Voltage stability analysis of radial distribution networks. Int J Electr Power Energy Syst 2001;23:129–35.
[126] Aman MM, Jasmon GB, Bakar AHA, Mokhlis H. Optimum network reconfiguration based on maximization of system loadability using continuation power flow theorem. Int J Electr Power Energy Syst 2014;54:123–33.
[127] Das D, Nagi HS, Kothari DP. Novel method for solving radial distribution networks. IEE Proc Gener Transm Distrib 1994;141:291–8.
[128] Eminoglu U, Hocaoglu MH. A new power flow method for radial distribution systems including voltage dependent load models. Electr Power Syst Res 2005;76:106–14.
[129] Chandramohan S, Atturulu N, Devi RPK, Venkatesh B. Operating cost minimization of a radial distribution system in a deregulated electricity market through reconfiguration using NSGA method. Int J Elec Power 2010;32:126–32.
[130] Thukaram D, Wijekoon Banda H, Jerome J. A robust three phase power flow algorithm for radial distribution systems. Electr Power Syst Res 1999;50:227–36.
[131] Kennedy J, Eberhart R. Particle swarm optimization. In: Neural networks, 1995 proceedings, IEEE international conference on 1995. p. 1942-8 vol.4.